Optimized suppression of coherent noise from seismic data using the Karhunen-Loève transform

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Signals obtained in land seismic surveys are usually contaminated with coherent noise, among which the ground roll (Rayleigh surface waves) is of major concern for it can severely degrade the quality of the information obtained from the seismic record. This paper presents an optimized filter based on the Karhunen-Loève transform for processing seismic images contaminated with ground roll. In this method, the contaminated region of the seismic record, to be processed by the filter, is selected in such way as to correspond to the maximum of a properly defined coherence index. The main advantages of the method are that the ground roll is suppressed with negligible distortion of the remnant reflection signals and that the filtering procedure can be automated. The image processing technique described in this study should also be relevant for other applications where coherent structures embedded in a complex spatiotemporal pattern need to be identified in a more refined way. In particular, it is argued that the method is appropriate for processing optical coherence tomography images whose quality is often degraded by coherent noise (speckle).

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I. INTRODUCTION

Locating oil reservoirs that are economically viable is one of the main problems in the petroleum industry. This task is primarily undertaken through seismic exploration, where explosive sources generate seismic waves whose reflections at the different geological layers are recorded at the ground or sea level by acoustic sensors (geophones or hydrophones). These seismic signals, which are later processed to reveal information about possible oil occurrence, are often contaminated by noise and properly cleaning the data is therefore of paramount importance [1]. Of particular concern is noise that shows coherence in space and time since it often appears stronger in magnitude than the reflection signal itself. In this context, the design of efficient filters for coherent noise not only is of great practical relevance but also remains a scientific challenge for which novel concepts and methods are required. An additional motivation to tackle this problem is that the filtering tools developed to treat this kind of noise may also find relevant applications in other physical problems where coherent structures embedded in a complex pattern need to identified properly. In particular, we shall argue below that the technique proposed in the present paper for processing seismic image can also be used to filter data acquired with other imaging technologies, such as optical coherence tomography.

In land seismic surveys, the seismic sources generate various type of surface waves which are regarded as noise since they do not contain information from the deeper subsurface. This so-called coherent noise represents a serious hurdle in the processing of the seismic data since it may overwhelm the reflection signal, thus severely degrading the quality of the information that can be obtained from the data. A source-generated noise of particular concern is the ground roll, which is the main type of coherent noise in land seismic records and is commonly much stronger in amplitude than the reflected signals. Ground rolls are surface waves whose vertical components are Rayleigh-type dispersive waves, with low frequency and low phase and group velocities.

An example of seismic data contaminated by ground roll is shown in Fig. 1. (The data shown in this figure were provided by the Brazilian Petroleum Company PETROBRAS.) This seismic section consists of land-based data with 96 traces (one for each geophone) and 1001 samples per trace. A typical trace is shown in Fig. 2 corresponding to geophone

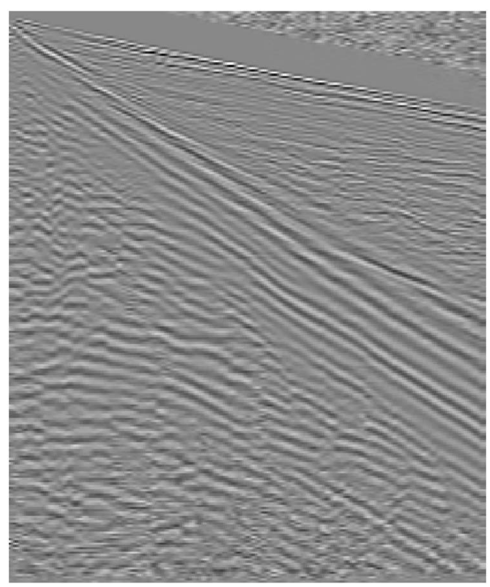

FIG. 1. A space-time plot of seismic data. The horizontal axis represents the offset distance and the vertical axis indicates time. The origin is at the upper left corner, and the maximum offset and time are 475 m and 1000 ms, respectively. The gray scale is such that black (white) corresponds to the minimum (maximum) amplitude of the seismic signal. The ground roll noise appears as downward oblique lines.

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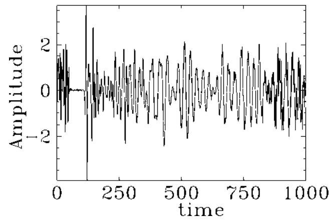

FIG. 2. Seismic signal recorded by a single geophone (trace 58). The amplitude is in arbitrary units and time in milliseconds.

58. The image shown in Fig. 1 was created from the 96 traces using a standard imaging technique. The horizontal axis in this figure corresponds to the offset distance between source and receiver and the vertical axis represents time, with the origin located at the upper left corner. The maximum offset is 475 m (the distance between geophones being 5 m) and the maximum time is 1000 ms. The gray levels in Fig. 1 change linearly from black to white as the amplitude of the seismic signal varies from minimum to maximum. Owing to its dispersive nature, the ground roll appears in a seismic image as a fanlike structure, which is clearly visible in Fig. 1.

Standard methods for suppressing ground roll include one-dimensional high-pass filtering and two-dimensional f-k filtering [1]. Such "global" filters are based on the elimination of specific frequencies and have the disadvantage that they also affect the uncontaminated part of the signal. Recently, "local" filters for suppressing the ground roll have been proposed using the Karhunen-Loève transform [2,3] and the wavelet transform [4,5]. The Wiener-Levinson algorithm has also been applied to extract the ground roll [6].

Filters based on the Karhunen-Loève (KL) transform are particularly interesting because of the *adaptivity* of the KL expansion, meaning that the original signal is decomposed in a basis that is obtained directly from the empirical data, unlike Fourier and wavelet transforms which use prescribed basis functions. The KL transform is a mathematical procedure (also known as proper orthogonal decomposition, empirical orthogonal function decomposition, principal component analysis, and singular value decomposition) whereby any complicated data set can be optimally decomposed into a finite, and often small, number of modes (called proper orthogonal modes, empirical orthogonal functions, principal components, or eigenimages) which are obtained from the eigenvectors of the data autocorrelation matrix. In applying the KL transform to suppress the ground roll, one must first map the contaminated region of the seismic record into a horizontal rectangular region. This transformed region is then decomposed with the KL transform and the first few principal components are removed to extract the coherent noise, after which the filtered data is inversely mapped back into the original seismic section. The advantage of this method is that the noise is suppressed with negligible distortion of the reflection signals, for only the data within the selected region are actually processed by the filter. Earlier versions of the KL filter [2,3] have, however, one serious drawback, namely, the fact that the region to be filtered must be picked by hand—a procedure that not only can be labor intensive but also relies on good judgment of the person performing the filtering.

In this paper we propose a significant improvement of the KL filtering method, in which the region to be filtered is selected automatically as an optimization procedure. We introduce the coherence index (CI), which gives a measure of the amount of energy contained in the most coherent modes for any given region of the data record. The optimal region is then chosen as that yields the maximum CI, thus ensuring that our KL filter removes the coherent noise in a most efficient way. Furthermore, introducing a quantitative criterion for selecting the "best" region to be filtered has the considerable advantage of yielding a largely unsupervised scheme for demarcating and efficiently suppressing the ground roll. It is also perhaps worth mentioning that our KL filter for coherent noise represents an unusual application of the KL transform, as this technique is normally used to eliminate noncoherent noisy components of a signal by disregarding higher-order modes in the KL expansion.

Although our main motivation here concerns the suppression of coherent noise in seismic data, we wish to emphasize that our method is applicable to other problems where one seeks to identify (and eventually remove) coherent structures embedded in a complex pattern. In particular, our optimized KL filter yields an image processing technique that is highly suitable for images where undesired coherent features degrade the quality of the information to be extracted from the image. One such instance is optical coherence tomography (OCT), where coherent noise (speckle) degrades the contrast of images of biological tissues, obscures region boundaries, and makes feature detection in OCT images a challenging problem [7]. In this context, our optimized KL filter may be used as an adaptive automated method for boundary detection in OCT images. Indeed, it is interesting to notice that other methods previously used in seismic signal processing, such as deconvolution techniques [8], have been successfully put to use in the enhancement of OCT images. Hence it is only natural to try to apply our method to this imaging technology. (We are currently working on this problem with OCT images of biological tissues [9] provided by Professor A. S. L. Gomes from our own department.)

The paper is organized as follows. In Sec. II we define the Karhunen-Loève transform and describe its main properties. In Sec. III we present the KL filter and an optimization procedure to select the noise-contaminated region to be parsed through the filter. The results of our optimized filter when applied to the data shown in Fig. 1 are presented in Sec. IV. Our main conclusions are summarized in Sec. V.

II. THE KARHUNEN-LOÈVE TRANSFORM

Consider a multichannel seismic data consisting of m traces with n samples per trace represented by an $m\times n$ matrix A, so that the element A_{ij} of the data matrix corresponds to the amplitude registered at the ith geophone at time j. For definiteness, let us assume that $m<n$, as is usually the case. We also assume for simplicity that the matrix A has full rank, i.e., $r=m$, where r denotes the rank of A. Letting the vectors $\vec{x}_i$ and $\vec{y}_j$ denote the elements of the ith row and the jth column of A, respectively, we can write

$$A = (\vec{y}_1\ \vec{y}_2\ \cdots\ \vec{y}_n) = \begin{pmatrix} \vec{x}_1 \\ \vec{x}_2 \\ \vdots \\ \vec{x}_m \end{pmatrix}. \tag{1}$$

With the above notation we have

$$A_{ij} = x_{ij} = y_{ji}, \tag{2}$$

where a_{ij} denotes the jth element of the vector $\vec{a}_i$. (To avoid risk of confusion matrix elements will always be denoted by capital letters, so that a lower-case symbol with two subscripts indicates vector elements.)

Next consider the following $m \times m$ symmetric matrix:

$$\Gamma \equiv AA^t, \tag{3}$$

where the superscript t denotes matrix transposition. It is a well-known fact from linear algebra that matrices of the form (3), also called covariance matrices, are positive definite [10]. Let us then arrange the eigenvalues λ_i of Γ in nonascending order, i.e., $\lambda_1 \geqslant \lambda_2 \geqslant \cdots \geqslant \lambda_m > 0$, and let $\vec{u}_i$ be the corresponding (normalized) eigenvectors.

The Karhunen-Loève transform of the data matrix A is defined as the $m \times n$ matrix Ψ given by

$$\Psi = U^t A, \tag{4}$$

where the columns of the matrix U are the eigenvectors of Γ:

$$U = (\vec{u}_1\ \vec{u}_2\ \cdots\ \vec{u}_m). \tag{5}$$

The original data can be recovered from the KL transform Ψ by the inverse relation

$$A = U\Psi. \tag{6}$$

We refer to this equation as the KL *expansion* of the data matrix A. To render such an expansion more explicit let us denote by the $\vec{\chi}_i$, $i=1,\ldots,m$, the elements of the ith row of the KL matrix Ψ, that is,

$$\Psi = \begin{pmatrix} \vec{\chi}_1 \\ \vec{\chi}_2 \\ \vdots \\ \vec{\chi}_m \end{pmatrix}. \tag{7}$$

Then (6) can be written as

$$A = \sum_{i=1}^{m} A_i = \sum_{i=1}^{m} \vec{u}_i \vec{\chi}_i, \tag{8}$$

where matrix multiplication is implied between the column vector $\vec{u}_i$ and the row vector $\vec{\chi}_i$. The eigenvectors $\vec{u}_i$ are called empirical eigenvectors, proper orthogonal modes, or KL modes, and we shall refer to the matrix $A_i = \vec{u}_i \vec{\chi}_i$ as the ith eigenimage of A. (This differs from the usual definition of eigenimages by a factor of $\sqrt{\lambda_i}$ but such a distinction is not relevant here.) In the context of principal component analysis, the eigenvectors $\vec{u}_i$ define the principal components whereas the row vectors $\vec{\chi}_i$ of the KL transform Ψ are said to contain the scores of the ith principal component.

It is usual to define the energy E of the data in matrix A as the sum of all eigenvalues of the matrix Γ,

$$E = \sum_{i=i}^{m} \lambda_i. \tag{9}$$

It then follows that λ_i can be interpreted as the energy captured by the ith empirical eigenvector $\vec{u}_i$ [11], so that the relative energy E_i contained in the ith KL mode is given by

$$E_i = \frac{\lambda_i}{\sum_{i=i}^{m} \lambda_i}. \tag{10}$$

We note furthermore that since Γ is a covariancelike matrix its eigenvalues λ_i can also be interpreted as the variance of the respective principal component $\vec{u}_i$. We thus say that the higher λ_i, the more coherent the KL mode $\vec{u}_i$ is. In this context, the most energetic modes are identified with the most coherent ones, and vice versa.

An important property of the KL expansion is that it is "optimal" in the following sense: if we form the matrix Ψ_k by keeping the first k rows of Ψ and setting the remaining $m-k$ rows to zero, then the matrix A_k given by

$$A_k = U\Psi_k \tag{11}$$

is the best approximation to A by a matrix of rank $k<m$ in the Frobenius norm (the square root of the sum of the squares of all matrix elements) [11]. This optimality property of the KL expansion lies at the heart of its applications in dimensionality reduction and data compression, for it allows to approximate the original data A by a smaller matrix A_k with minimum loss of information (in the above sense). Another interpretation of relation (11) is that it gives a low-pass filter [12], for in this case only the first k KL modes are retained in the filtered data A_k.

On the other hand, if the relevant signal in the application at hand is contaminated with coherent noise, as is the case of the ground roll in seismic data, one can use the KL transform to remove efficiently such noise by constructing a high-pass filter. Indeed, if we form the matrix Ψ_k' by setting to zero the first k rows of Ψ and keeping the remaining ones, then the matrix A'_k given by

$$A_k' = U\Psi_k' \tag{12}$$

is a filtered version of A where the first k "most coherent" modes have been removed. However, if the noise is localized in space and time it is then best to apply the filter only to the contaminated part of the signal. In previous versions of the KL filter the choice of the region to be parsed through the filter is made *a priori*, according to the best judgment of the person carrying out the filtering, thus lending a considerable degree of subjectivity to the process. In the next section, we will show how one can use the KL expansion to implement an automated filter where the undesirable coherent structure can be optimally identified and removed.

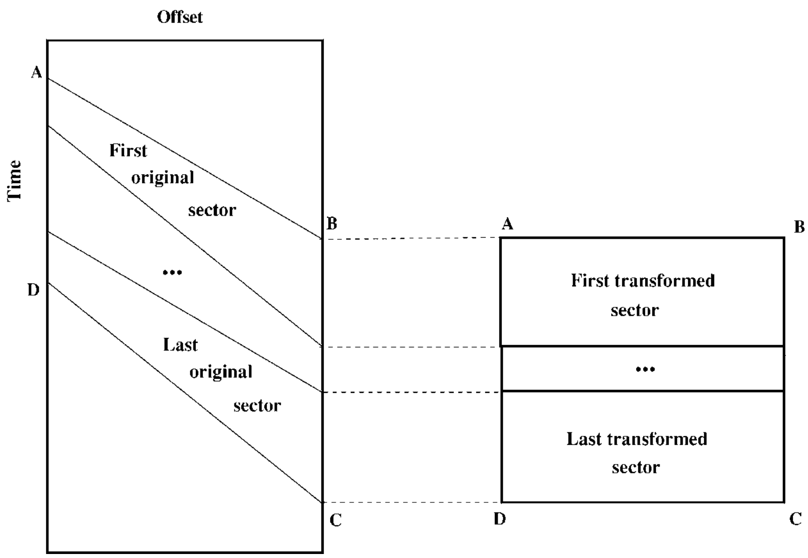

FIG. 3. Schematic diagram for demarcating the ground roll on a seismic section and the corresponding rectangular sectors obtained by applying a linear map.

III. THE OPTIMIZED KL FILTER

As already mentioned, owing to its dispersive nature the ground-roll noise appears in a seismic image as a typical fanlike coherent structure. This space-time localization of the ground roll allows us to apply a sort of surgical procedure to suppress the noise, leaving intact the uncontaminated region. To do that, we first pick lines to demarcate the start and end of the ground roll and, if necessary, intermediate lines to demarcate different wavetrains, as indicated schematically in Fig. 3. In this figure we have for simplicity used straight lines to demarcate the sectors but more general alignment functions, such as segmented straight lines, can also be chosen [2,3]. To make our discussion as general as possible, let us assume that we have a set of N parameters $\{\theta_i\}$, $i=1,\ldots,N$, describing our alignment functions. For instance, in Fig. 3 the parameters $\{\theta_i\}$ would correspond to the coefficients of the straight lines defining each sector.

Once the region contaminated by the ground roll has been demarcated, we map each sector onto a horizontal rectangular region by shifting and stretching along the time axis; see Fig. 3. The data points between the top and bottom lines in each sector are mapped into the corresponding new rectangular domain, with the mapping being carried out via a cubic convolution interpolation technique [13]. After this alignment procedure the ground-roll events will become approximately horizontal, favoring its decomposition in a smaller space. Since any given transformed sector has a rectangular shape it can be represented by a matrix, which in turn can be decomposed in empirical orthogonal modes (eigenimages) using the KL transform. The first few modes, which contain most of the ground roll, are then subtracted to extract the coherent noise. The resulting data for each transformed sector are finally subjected to the corresponding inverse mapping to compensate for the original forward mapping. This leaves the uncontaminated data (lying outside the demarcated sectors) unaffected by the whole filtering procedure.

The KL filter described above has indeed shown good performance in suppressing source-generated noise from seismic data [2,3]. The method has, however, the drawback that the region to be filtered must be picked by hand, which renders the analysis somewhat subjective. In order to overcome this difficulty, it would be desirable to have a quantitative criterion based on which one could decide what is the best choice for the parameters $\{\theta_i\}$ describing the alignment functions. In what follows, we propose an optimization procedure whereby the region to be filtered can be selected automatically, once the generic form of the alignment functions is prescribed.

Suppose we have chosen l sectors to demarcate the different wave trains in the contaminated region of the original data, and let $\{\theta_1,\ldots,\theta_N\}$ be the set of parameters characterizing the respective alignment functions that define these sectors. Let us denote by $\tilde{A}_k$, $k=1,\ldots,l$, the matrix representing the kth transformed sector obtained from the linear mapping of the respective original sector, as discussed above. For each transformed sector $\tilde{A}_k$ we then compute its KL transform and calculate the *coherence index* C_k for this sector, defined as the relative energy contained in its first KL mode:

$$C_k = \frac{\lambda_1^k}{\sum_{i=1}^{r_k} \lambda_i^k}, \qquad (13)$$

where λ_i^k are the eigenvalues of the correlation matrix $\tilde{\Gamma}_k = \tilde{A}_k \tilde{A}_k^t$ and r_k is the rank of $\tilde{A}_k$. As defined above, C_k represents the relative weight of the most coherent mode in the KL expansion of the transformed sector $\tilde{A}_k$. (A quantity analogous to our CI is known in the oceanography literature as the similarity index [14].)

Next we introduce an overall coherence index $C(\theta_1,\ldots,\theta_N)$ for the entire demarcated region, defined as the average coherence index of all sectors:

$$C(\theta_1,\ldots,\theta_N) = \frac{1}{l}\sum_{k=1}^{l} C_k. \qquad (14)$$

As the name suggests, the coherence index is a measure of the amount of coherent energy contained in the chosen demarcated region given by the parameters $\{\theta_i\}_{i=1}^N$. Thus, the higher C the larger the energy contained in the most coherent modes in that region. For the purpose of filtering coherent noise it is therefore mostly favorable to pick the region with the largest possible C. We thus propose the following criterion to select the optimal region to be filtered: vary the parameters $\{\theta_i\}$ over some appropriate range and then choose the values θ_i^* that maximize the coherence index C, that is,

$$C(\theta_1^*, \ldots, \theta_N^*) = \max_{\{\theta_i\}}[C(\theta_1, \ldots, \theta_N)]. \qquad (15)$$

Once we have selected the optimal region, given by the parameters $\{\theta_i^*\}_{i=1}^N$, we then simply apply the KL filter to this region as already discussed: we remove the first few eigenimages from each transformed sector and inversely map the data back into the original sectors, so as to obtain the final filtered image. In the next section we will apply our optimized KL filtering procedure to the seismic data shown in Fig. 1.

IV. RESULTS

Here we illustrate how our optimized KL filter works by applying it to the seismic data shown in Fig. 1. In this case, it suffices to choose only one sector to demarcate the entire region contaminated by the ground roll. This means that we have to prescribe only two alignment functions, corresponding to the uppermost and lowermost straight lines (lines AB and CD, respectively) in Fig. 3. To reduce further the number of free parameters in the problem, let us keep the leftmost point of the upper line (point A in Fig. 3) fixed to the origin, so that the coordinates (i_A, j_A) of point A are set to (0, 0), while allowing the point B to move freely up or down within certain range; see below. Similarly, we shall keep the rightmost point of the lower line (point C in Fig. 3) pinned at a point (i_C, j_C), where $i_C=95$ and j_C is chosen so that the entire ground-roll wave train is above point C. The other end point of the lower demarcation line (point D in Fig. 3) is allowed to vary freely. With such restrictions, we are left with only two free parameters, namely, the angles θ_1 and θ_2 that the upper and lower demarcation lines make with the horizontal axis. So reducing the dimensionality of our parameter space allows us to visualize the coherence index $C(\theta_1, \theta_2)$ as a two-dimensional (2D) surface. For the case in hand, it is more convenient, however, to express CI not as a function of the angles θ_1 and θ_2 but in terms of two other new parameters introduced below.

Let the coordinates of point B, which defines the right end point of the upper demarcation line in Fig. 3, be given by (i_B, j_B), where $i_B=95$. In our optimization procedure we let point B move along the right edge of the seismic section by

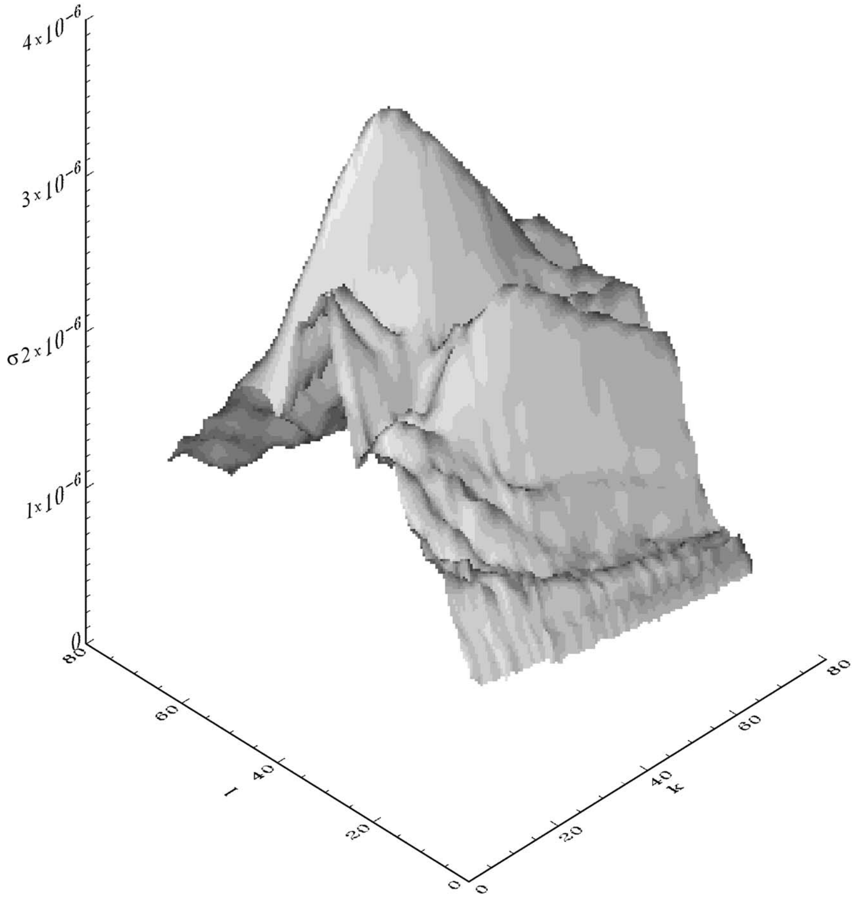

FIG. 4. The coherence index as a function of the indices k and l that define the demarcation lines; see text.

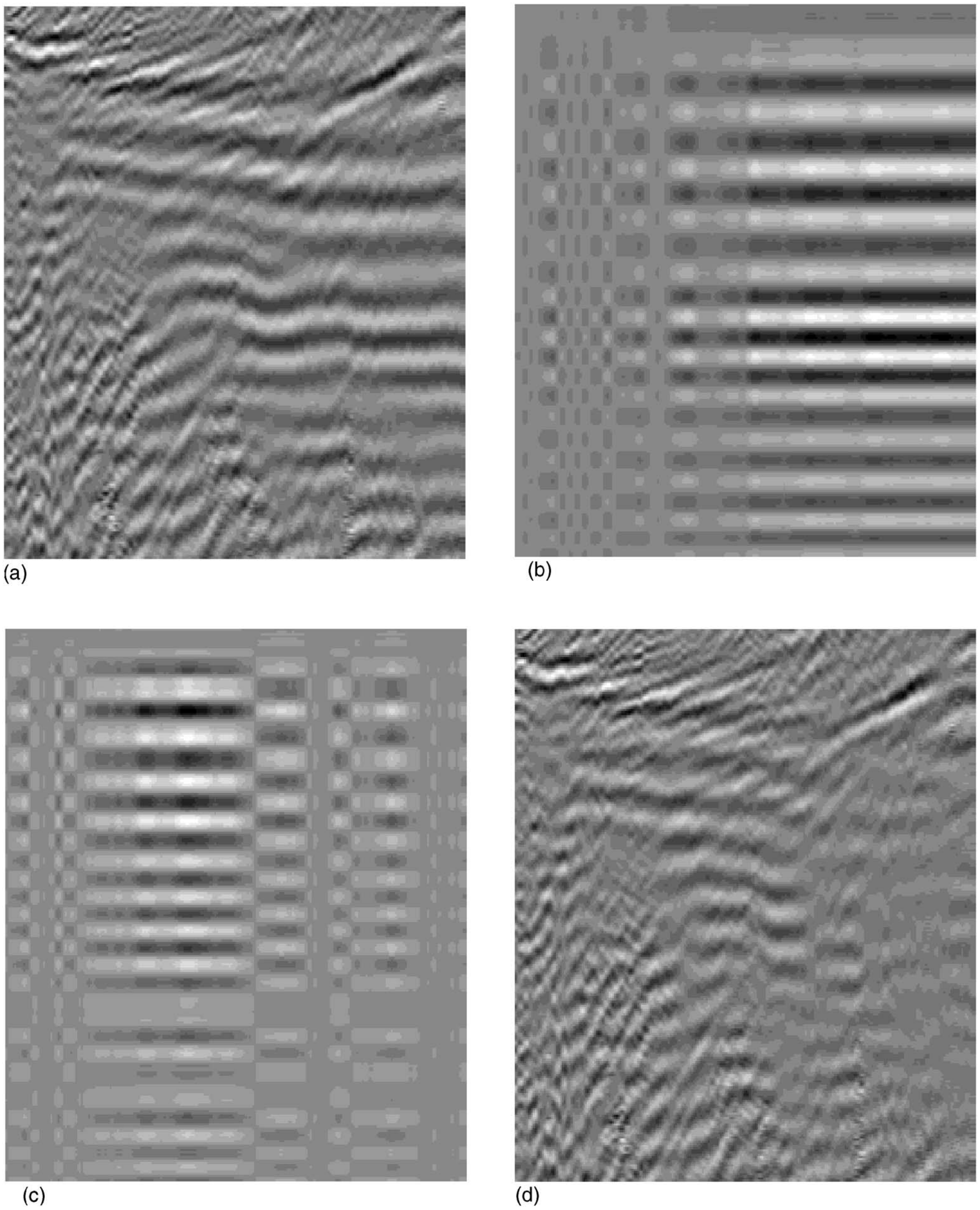

FIG. 5. (a) The selected region in the new domain; (b) its first eigenimage; (c) the second eigenimage; and (d) the result after subtracting the first eigenimage.

allowing the coordinate j_B to vary from a minimum value $j_{B_{min}}$ to a maximum value $j_{B_{max}}$, so that we can write

$$j_B = j_{B_{min}} + k\Delta_B, \quad k = 0, 1, \ldots, N_B, \tag{16}$$

where N_B is the number of intermediate sampling points between $j_{B_{min}}$ and $j_{B_{max}}$, and $\Delta_B = (j_{B_{max}} - j_{B_{min}})/N_B$. Similarly, for the coordinates (i_D, j_D) of point D in Fig. 3, which is the moving end point of the lower straight line, we have $i_D = 0$ and

$$j_D = j_{D_{min}} + l\Delta_D, \quad l = 0, 1, \ldots, N_D, \tag{17}$$

where N_D is the number of sampling points between $j_{D_{min}}$ and $j_{D_{max}}$, and $\Delta_D = (j_{D_{max}} - j_{D_{min}})/N_D$.

For each choice of k and l in Eqs. (16) and (17), we apply the procedure described in the previous section and obtain the coherence index $C(k,l)$ of the corresponding region. In Fig. 4 we show the energy surface $C(k,l)$, for the case in which $j_{B_{min}} = 280$, $j_{B_{max}} = 600$, $j_C = 864$, $j_{D_{min}} = 0$, $j_{D_{max}} = 576$, and $N_B = N_D = 64$. We see in this figure that C possesses a sharp peak, thus showing that this criterion is indeed quite discriminating with respect to the positioning of the lines demarcating the region contaminated by the ground roll. The global maximum of C in Fig. 4 is located at $k = 27$ and $l = 45$, and in Fig. 5(a) we show the transformed sector obtained from the linear mapping of this optimal region. In this figure one clearly sees that the ground-roll wave trains appear mostly as horizontal events. In Fig. 5(b) we present the first eigenimage of the data shown in Fig. 5(a), which corre-

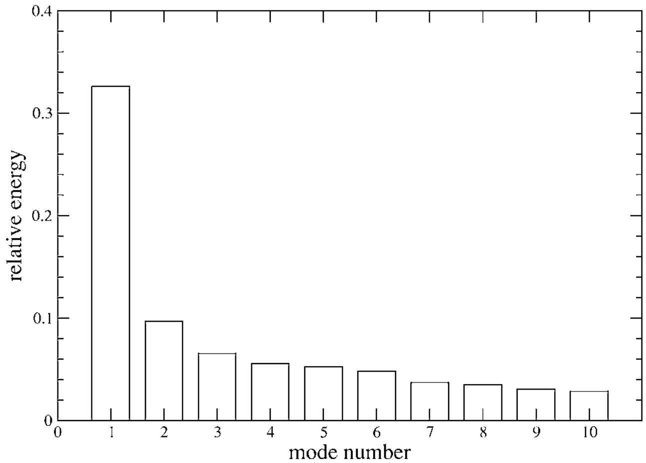

FIG. 6. The relative energy of the first ten KL modes of the region shown in Fig. 5(a).

sponds to about 33% of the total energy of the image in Fig. 5(a), as can be seen in Fig. 6 where we plot the relative energy E_i captured by the first ten eigenimages. The second eigenimage, shown in Fig. 5(c), captures about 10% of the total energy, with each successively higher mode contributing successively less to the total energy; see Fig. 6. In Fig. 5(d) we give the result of removing the first KL mode [Fig. 5(b)] from Fig. 5(a). It is clear in Fig. 5(d) that by removing only the first eigenimage the main horizontal events (corresponding to the ground roll) have already been greatly suppressed.

Performing the inverse mapping of the image shown in Fig. 5(c) yields the data seen in the region between the two white lines in Fig. 7(a), which shows the final filtered image for this case (i.e., after removing the first KL mode from the transformed region). We see that the ground roll inside the demarcated region in Fig. 7(a) has been considerably suppressed, while the uncontaminated signal (lying outside the marked region) has not been affected at all by the filtering procedure. If one wishes to filter further the ground-roll noise one may subtract successively higher modes. For example, in Fig. 7(b) we show the filtered image after we also subtract the second eigenimage. One sees that there is some minor improvement, but removing additional modes is not recommended for it starts to degrade the relevant signal as well.

V. CONCLUSIONS

An optimized filter based on the Karhunen-Loève transform has been constructed for processing seismic data contaminated with coherent noise (ground roll). A great advantage of the KL filter lies in its local nature, meaning that only the contaminated region of the seismic record is processed by the filter, which allows the ground roll to be removed without distorting most of the reflection signal. Another advantage is that it is an adaptive method in the sense the signal is decomposed in an empirical basis obtained from the data themselves. We have improved considerably the KL filter by introducing an optimization procedure whereby the ground roll region is selected so as to maximize an appropriately defined coherence index. We emphasize that our method requires as input only the generic alignment functions to be used in the optimization procedure as well as the number of eigenimages to be removed from the selected region. These may vary depending on the specific application at hand. However, once these choices are made, the filtering task can proceed in the computer in an automated way.

Although our main motivation here has been suppressing coherent noise from seismic data, our method is by no means

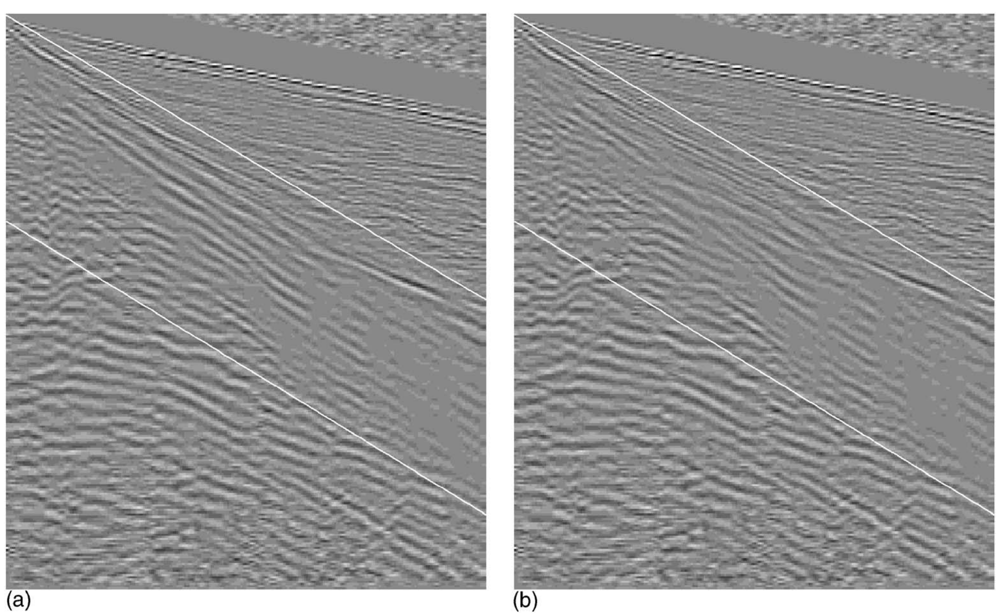

FIG. 7. (a) The filtered seismic section after removing the first eigenimage of the selected region shown in Fig. 5(a). In (b) we show the result after removing the first two eigenimages.

restricted to geophysical applications. In fact, the KL filter described in this paper represents an image processing technique that is quite suitable to process images where coherent noise degrades image quality. One such case is the image-degrading effects of speckle in optical coherence tomography. We are currently testing our KL filter as a method for enhancing OCT images and hope to report on this interesting problem in future presentations.

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